**Mathematics 503 Complex Analysis Fall 2017**

**Week 6 workshop problems**

1. Show that the polynomial $p(z) = z^n(z-2) - 1$ has n distinct roots in the unit disk and a real root larger than 1.

2. Let $P(x)/Q(x)$ be a ratio of polynomials such that the degree of the denominator is at least one more than the degree of the numerator and the denominator has no real zeros. Show that

$$\int_{-\infty}^{\infty} e^{ix} P(x)/Q(x)\, dx = 2\pi i \sum Res_{z_i}(e^{iz} P(z)/Q(z))$$

   where the sum is taken over the zeros $z_i$ of $Q(z)$ in the upper half plane. Hint: use for a contour a rectangle with base $[-A, B]$ and top segment joining $-A + iY, B + iY$ and let $A, B, Y$ approach infinity.

3. Let $f(z)$ be holomorphic on an open set containing $\{z||z| \leq 1\}$ and suppose $|f(z)| < 1$ on the unit circle. Show that $f(z) = z^n$ has $n$ solutions in the open unit disk. Show that $f(z)$ has a unique fixed point in the unit disk.

4. Compute

$$\int_0^{\infty} \frac{x}{x^4 + 1}\, dx.$$