

Week 5 workshop problems

1. Find the maximum of $|\sin(z)|$ on the closed unit disk.
2. Compute $\int_0^\infty 1/(x^4 + 1) dx$.
3. Show that the Fourier transforms of $f(x) = 1/(1 + x^2)^2$ is

$$\hat{f}(y) = \frac{\pi}{2}(1 + 2\pi|y|)e^{-2\pi|y|}.$$

4. (L'Hospital's rule) Show that if f, g are holomorphic in a neighborhood of a point $z_0 \in \mathbb{C}$ and each has a zero of order k there then

$$\lim_{z \rightarrow z_0} \frac{f(z)}{g(z)} = \frac{f^{(k)}(z_0)}{g^{(k)}(z_0)}$$