

## Week 12 workshop problems

1. Show that

$$\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}.$$

2. Let  $f(z)$  be an entire function and  $n$  a positive integer. Show that  $f(z) = g(z)^n$  for some entire function  $g(z)$  if and only if the multiplicity of all zeros of  $f(z)$  is divisible by  $n$ .
3. Show that if  $|z| < 1$  then

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\cdots = \frac{1}{1-z}$$