Mathematics 435 Geometry Fall 2021

REVIEW QUESTIONS for EXAMINATION II

The questions below cover material covered in Mathematics 435 in weeks 6 through 11 this semester. The second exam will cover similar topics. If you wish to discuss these come by office hours or ask in class on 11/22/21.

1. Does there exist a linear fractional transformation taking the real numbers $0, 1, 2, 3$ respectively to $3, 4, 5, 7$? Either give the transformation, or prove that one does not exist. Same question for the points $2, 0, 1, 8$ and $-4/3, 1, 0, \infty$.

2. Find an isometry of the Euclidean plane which fixes the bisector of an angle and interchanges the sides of the angle. Show that any point on the angle bisector is equidistant from the two rays forming the angle. Show that the bisectors of angles in a triangle meet in a common point.

3. Suppose three circles are given, each containing the other two in the exterior. Take each pair of circles and draw lines which are tangent to each of the circles in the pair but not crossing the segment joining their centers. Show that the intersections of the common tangents of pairs of tangents are collinear. Hint: Draw a picture. Let A, B, C be the centers of the circles and A', B', C' be the intersections of pairs of tangents which are outside the triangle ABC. Let P be the intersection of the angle bisectors of $A'B'C'$. Show that projection from P maps the vertices of ABC to those of $A'B'C'$. Now apply one of the theorems of projective geometry.

3'. State Pappus theorem about incidences related to 6 points on two lines in the projective plane. Prove Pappus theorem by applying a suitable transformation of the projective plane to reduce it to a configuration having several pairs of parallel lines.

4. A projective transformation L of the real projective plane maps points with homogenous coordinates $[1, 0, 0], [0, 1, 0], [0, 0, 1], [1, 1, 1]$ to $[-1, 2, 0], [1, 3, 1], [0, 2, 3], [-3, -2, 1]$. What is the image of the point $[3, 4, 5]$ under L.

5. Find the equation of the line in the real projective plane containing the points with homogeneous coordinates $[1, 2, 3]$ and $[4, 5, 6]$.

6. Let V be a real vector space. Define the projectivization $\mathbf{P}(V)$ of the vector space. Describe the points and lines in $P(V)$.

7. A picture of a straight road shows a Ford Mustang following a VW Beetle in the same lane. In the picture the Mustang measures 2 inches and the VW 1 inch, and the space

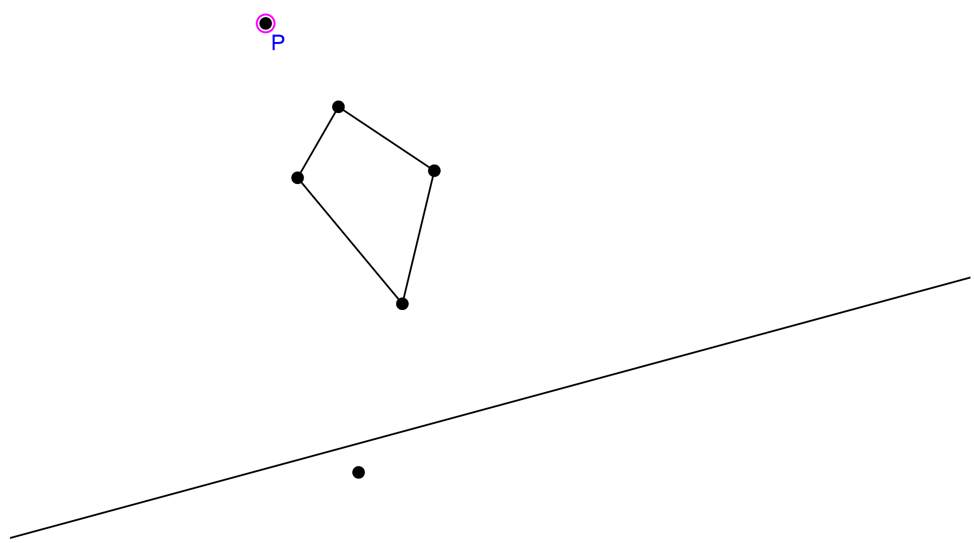
between 3 inches. Mustangs are 15 feet long and Beetles are 13 feet long. How many feet behind the Beetle was the Mustang?

8. Consider the set of points (x, y) in the Euclidean plane satisfying

$$4x^2 - 2xy + 5y^2 - 7x - 2y - 15 = 0.$$

Is this figure a parabola, an ellipse, or a hyperbola? Describe this figure as a translate of a rotation of a standard conic section. (See the supplement on conics)

9. The image below shows a light source at P, the image of a corner of the kite shadow on the ground plane, and the line of intersection of the plane of the kite and the ground plane. Draw the shadow of the kite on the ground, explaining carefully how Desargues theorem is used.



10. A spherical triangle has angles $\pi/2, \pi/2, \pi/3$. Find the area of this triangle.

11. Suppose that f is a rotation of 90 degrees of the unit sphere centered at the origin in $\mathbf{R}^3$ about the line through the origin containing the point (1,1,1) and g is a rotation of 90 degrees about the line through the origin containing the point (1,0,1). The composition gf is a rotation of the sphere. Find a nonzero point on the axis of the rotation gf. (note revised 11/22/21 10:24 PM)