Mathematics 356 Theory Of Numbers Fall 2018

SAMPLE EXAMINATION I

No notes, books, cellphones or electronic devices
Calculators may NOT BE USED
Show all of your work, justifying all steps

1.(10 points) Use the Dirichlet Box Principle to show that in any collection of 2018 distinct positive integers there are two with difference divisible by 1000.

2. (14 points)

a) (7 points)Find a pair of integers (x,y) solving $73x + 101y = 3$.

b) (7 points) Find all integer solutions to the equation $73x + 101y = 3$.

3.(14 points)

a) (7 points) Find the multiplicative inverse of 43 modulo 219

b) (7 points) Find all solutions z to the linear congruence $43z \equiv 9 \pmod{219}$.

4.(16 points) When eggs are removed from a basket 5 at a time there remain 4 eggs. The eggs are replaced in the basket, and then removed 6 at a time leaving 5 remaining, and when they are all placed back in the basket and removed 7 at a time there are none remaining in the basket. What is the least number of eggs that could have originally been in the basket?

5.(12 points) Recall the least common multiple $[m, n]$ of two positive integers is the minimum positive number divisible by m, n. Use the Fundamental Theorem of Arithmetic to find how many ordered pairs of positive integers have least common multiple $2 \cdot 3^2$.

6.(12 points) Show by induction that

$$1 \cdot 1! + 2 \cdot 2! + \cdots n \cdot n! = (n+1)! - 1$$

where $k! = 1 \cdot 2 \cdot 3 \cdots (k-1) \cdot k$ is the factorial function.

7. (10 points) Compute the remainder of 5^{129} after dividing by 14.

8. (12 points) Find integers x, y satisfying

$$\begin{aligned} 7x + 3y &\equiv 1 \pmod{31} \\ 11x + 5y &\equiv 2 \pmod{31} \end{aligned}$$