

Formulas you should know and understand for Exam 1
There will be NO FORMULA SHEET allowed at the exam

$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta, \quad |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$$

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}} \mathbf{b}$$

$$\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{v^3} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{\left| \frac{d\mathbf{T}}{dt} \right|}{v}$$

$$D_{\mathbf{u}} f(x_0, y_0) = \nabla f(x_0, y_0) \bullet \mathbf{u}$$

$$z = f(x, y), x = g(t), y = h(t) \text{ then } \frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

Know how to compute dot products of vectors in n-space, angles between vectors and cross product of vectors in 3-space.

Know that the velocity vector for a parametric curve $\mathbf{r}(u)$ at the point labeled by u is $\mathbf{r}'(u)$, the speed $v(u)$ is the length of $\mathbf{r}'(u)$, and the unit tangent vector is $\mathbf{T}(u) = \mathbf{r}'(u)/|\mathbf{r}'(u)|$.

The arclength of a parametric curve $\mathbf{r}(u)$ as the parameter varies from a to b is $\int_a^b |\mathbf{r}'(u)| du$

Chain rule: If $x_i(t_1, \dots, t_m)$ are differentiable functions of m-variables and F is a differentiable real valued function of n-variables, and then

$$\partial/\partial t_i F(x_1, \dots, x_n) = \nabla f(x_1(t_1, \dots, t_m), \dots, x_n(t_1, \dots, t_m)) \cdot (\partial x_1/\partial t_m, \dots, \partial x_n/\partial t_m).$$

In particular $\partial/\partial t f(\mathbf{r}(t)) = \nabla f(\mathbf{r}(t)) \cdot \mathbf{r}'(t)$.

The plane through point (x_0, y_0, z_0) perpendicular to vector n is the set of (x, y, z) such that

$$n \cdot ((x, y, z) - (x_0, y_0, z_0)) = 0$$