# Homework 6 (Due 3/3/2017)

Math 622

March 3, 2017

1. Shreve Exercise 7.7
2. Let $S_t$ follows the Black-Scholes mode:

$$dS_t = rS_t dt + \sigma S_t d\tilde{W}_t.$$

Find the PDE for **up and in** Put option with barrier $L$ and strike $K$. That is

$$V_T = (K - S_T)^+ \mathbf{1}_{\{\max_{0 \le u \le T} S_u > L\}},$$

where $L < K$.

3. Let $S_t$ follows the Black-Scholes mode:

$$dS_t = rS_t dt + \sigma S_t d\tilde{W}_t.$$

where

$$S_0 = 54.5, \sigma = 0.2, r = 0.05.$$

Consider the up and out call option

$$V_T = (S_T - K)^+ \mathbf{1}_{\{\max_{0 \le u \le T} S_u \le L\}},$$

where $L = 55, K = 45, T = 2$.

a) Use Monte-Carlo simulation to find $V_0$.

b) Because $S_0$ is very close to $L$ and $T$ is not small, there will be many sample paths in part a) expiring out of the money by crossing over the barrier. This makes the Monte-Carlo simulation less efficient. One possible idea is using Girsanov theorem (essentially Importance Sampling in spirit) to change the drift of the Brownian motion to a negative number to have more sample paths converging in the money. Write down such a scheme and the pricing formula under the change of measure.

c) Implement your scheme in part b). Is it more efficient than the Monte-Carlo in part a) ?

4. Let $S_t$ follows the Black-Scholes mode:

$$dS_t = rS_t dt + \sigma S_t d\tilde{W}_t.$$

where

$$\sigma = 0.2, r = 0.05.$$

Consider a near expiry up and out call option

$$V_T = (S_T - K)^+ \mathbf{1}_{\{\max_{0 \le u \le T} S_u \le L\}},$$

where $L = 55, K = 45, T = 0.1$. In this problem we want to estimate the Delta of this option when $S_0 = 54.5$.

a) Use Monte-Carlo simulation to find $V_0$ when $S_0 = 54.5$.

b) Use Monte-Carlo simulation to find $V_0$ when $S_0 = 54.51$.

c) Use your answers in a) and b) to estimate the Delta of the option when $S_0 = 54.5$. Is it small or large?

d) Repeat the calculations for $S_0 = 54.95, S_0 = 54.951$ and $T = 0.05$.

Remark: We expect Delta to be large in this case. One generally has to take care about the numerical techniques near expiry and near the barrier. To obtain an accurate estimate of Delta you might need to do a large number of simulations. Alternatively, you might want to use other techniques to accelerate convergence, such as Importance sampling in the previous question, or some other methods of your choice. Compare your answers with your friends to make sure you get a reasonable result.

5. Let $S_t$ follows the Black-Scholes mode:

$$dS_t = rS_t dt + \sigma S_t d\tilde{W}_t.$$

where

$$r = 0.05.$$

Consider the Asian call option

$$V_T = (\int_0^T S_u du - K)^+,$$

where $S_0 = 50, K = 45, T = 1$. In this problem we want to compare the Asian option with the European call option, especially when volatility is high.

a) Use Monte-Carlo simulation to compute the price of the Asian call option with $\sigma = 0.1$. Compare it with the price of European call option with the same parameters. Which one is higher? Can you give an intuition why?

b) Use Monte-Carlo simulation to compute the Vega (partial with respect to $\sigma$ ) of the Asian call option at $\sigma = 0.1$. Compare it with the Vega of European call option with the same parameters. Which one is higher? Can you give an intuition why?

c) Use Monte-Carlo simulation to compute the price of the Asian call option with $\sigma = 1$. Compare it with the price of European call option with the same parameters. Which one is higher? Can you give an intuition why?

d) Use Monte-Carlo simulation to compute the Vega of the Asian call option with $\sigma = 1$. Compare it with the Vega of European call option with the same parameters. Which one is higher? Can you give an intuition why?