Homework 4 (Due 2/17/2017)

Math 622

March 3, 2017

In this homework there will be some Monte-Carlo simulation. Report all your codes with the homework.

1. Simulating Hull-White model: consider the Hull-White model for the short rate

$$dr_t = k(\mu - r_t)dt + \sigma d\tilde{W}_t,$$

where $\mu = 0.035, k = 2, \sigma = 0.05, r_0 = 0.01$. The price of zero-coupon bond via risk neutral pricing is

$$B(0,T) = \tilde{E}(e^{-\int_0^T r_s} ds).$$

a) Let $T = 10$. Find $B(0,T)$ by Monte-Carlo simulation (you can choose the appropriate number of simulations and the time step).

b) Print a sample path of r_t. Do you observe the mean reverting behavior to μ ?

c) The model is an affine-yield model :

$$B(0,T) = e^{-A(0,T) - C(0,T)r_0},$$

where $A(t,T), C(t,T)$ satisfies two ODEs that we derived. Shreve gave the explicit solutions to $A(t,T), C(t,T)$ in formulas (6.5.10), (6.5.11). Note that his model is slightly different from us in the drift term. Plug in the parameters and obtain $B(0,T)$ under this explicit formula. Compare the answer you derived in part a) with this explicit solution. Report the error term.

2. Simulating CIR model: consider the CIR model for the short rate

$$dr_t = k(\mu - r_t)dt + \sigma\sqrt{r_t}d\tilde{W}_t,$$

where $\mu = 0.035, k = 2, \sigma = 0.2, r_0 = 0.01$. The price of zero-coupon bond via risk neutral pricing is

$$B(0,T) = \tilde{E}(e^{-\int_0^T r_s}ds).$$

a) Let $T = 10$. Find $B(0,T)$ by Monte-Carlo simulation (you can choose the appropriate number of simulations and the time step).

b) Print a sample path of r_t. Do you observe the mean reverting behavior to μ ?

c) The model is an affine-yield model :

$$B(0,T) = e^{-A(0,T)-C(0,T)r_0},$$

where $A(t,T), C(t,T)$ satisfies two ODEs that we derived. Shreve gave the explicit solutions to $A(t,T), C(t,T)$ in formulas (6.5.16), (6.5.17). Note that his model is slightly different from us in the drift term. Plug in the parameters and obtain $B(0,T)$ under this explicit formula. Compare the answer you derived in part a) with this explicit solution. Report the error term.

3. In this problem you will need to simulate correlated Brownian motions. For an instruction to do so, see for example this link. Consider the Heston model for stochastic volatility:

$$\begin{aligned} dS_t &= rS_t dt + \sqrt{\sigma_t} S_t d\widetilde{W}_t^1 \\ d\sigma_t &= k(\mu - \sigma_t)dt + \gamma\sqrt{\sigma_t} d\widetilde{W^2}_t, \end{aligned}$$

where $d\langle \widetilde{W}^1, \widetilde{W}^2 \rangle_t = \rho dt$ and

$$S_0 = 50, \sigma_0 = 0.2, r = 0.01, \mu = 0.15, k = 2, \gamma = 0.1, \rho = 0.5.$$

The risk neutral pricing formula for call option based on Heston model is

$$V_0 = \widetilde{E}(e^{-rT}(S_T - K)^+).$$

Let $T = 1$ in this problem.

a) Let $K = 55$. Use Monte-Carlo simulation to find V_0. Compare your answer with Black-Scholes price (where clearly we just use $\sigma = \sqrt{\sigma_0} = \sqrt{0.2}$). Is Heston price higher or lower than Black-Scholes price? Can you give an intuition on why ?

b) Repeat part a) with $\rho = -0.5$. Compare this answer with part a) answer where $\rho = 0.5$. Is it higher or lower? Can you give an intuition on why ?

4. Testing the affine yield hypothesis : there are many articles written that discuss the affine yield model in terms of fitting empirical data. Two particular articles are:

a. Testing linearity in term structures by Chiara Peroni.

b. Estimating and testing exponential-affine term structure models by Kalman filter by JC Duan, JG Simonato.

Both articles are available via Rutgers library and attached in this homework's email. The first article is probably easier to read but has only 1 citation. The second has a lot of citations but may be more techinically challenging. For this problem, you can choose either one of the articles and write a short report on :

(i) The methodology of the article (the specific steps as if you will have to implement the method for a work project)

(ii) The conclusion of the article (is the affine yield model appropriate ?)

(iii) Do you agree or disagree with the article?