Homework 3 (Due 2/10/2017)

Math 622

March 3, 2017

This homework is a mini project. In class, we discuss that the forward-future spread is :

$$For_S(0, T) - Fut_S(0, T) = \frac{1}{B(0, T)} \widetilde{Cov}(D(T), S(T)),$$

where $D(T) = e^{-\int_0^T r_s ds}$ is the discounting process. There are two things we can learn from this formula:

a. The sign of the spread tells us about the sign of the covariance of the underlying and the short rate.

b. The magnitude of the spread (may?) tells us about the magnitude of the short rate in the future interval $[0, T]$.

Note that in a sense, the spread gives a "implied" covariance of the future discounting process $D(T)$ and the future price of the underlying $S(T)$. A concept similar to implied volatility being derived from fitting Black-Scholes price to market data. In this homework, I'd like you to do the following:

1. Find 3 assets that respectively have negative, positive and (close to) zero forward-future spread (preferably something common like a commodity index). This can be the current spread or the spread at some point in history. For reference let's call that point t_0 (say May 1st, 2008).

2. Check using historical data before t_0 to verify if the correlation between the asset price and the short rate follows the expected result in the above formula. In some sense this is not quite what the formula says because we are checking the historial covariance before t_0, not the "future" covariance.

3. Check using historical data after t_0 to verify if the correlation between the asset price and the short rate follows the expected result in the above formula.

4. Pick a period in history when the short rate rises or falls sharply (maybe right now is a good time?). Can you match the magnitude of the spread with the short rate magnitude after in some way?

This is a very open-ended assignment. If you could provide some decent data for questions 1, 2 and 3 you will receive full credit for the homework. Feel free to re-interpret the questions into some way you think that may make more sense. If you do so, explain why you want to phrase the question in such way.

If you want to investigate more into this topic you can talk to me for a possible extra credit project that can be applied to the final grade.