Homework 7 (Due 3/3/2017)

Math 622

April 7, 2017

1. Let (X, Y) have joint Normal distribution with mean $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and covariance matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (which implies that X, Y are independent). Generate 100 sample points of (X, Y).

a) Use the methods in Longstaff and Schwartz to compute $E(X|Y)$ with basis functions $1, y, y^2$. Compare your answer with the theoretical answer (which is just $E(X)$ in this case).

b) Use the methods in Longstaff and Schwartz to compute $E(X|Y)$ with the Laguerre basis functions (first three) as described in page 122. Compare with the theoretical answer and the answer in part a)

c) (Optional) If your answer in a) or b) are not close enough, either increase the number of basis functions or the sample size to obtain satisfactory accuracy.

2. Let (X, Y) have joint Normal distribution with mean $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and covariance matrix $\begin{bmatrix} 4 & 3 \\ 3 & 9 \end{bmatrix}$. Generate 100 sample points of (X, Y).

a) Use the methods in Longstaff and Schwartz to compute $E(X|Y)$ with basis functions $1, y, y^2$. Compare your answer with the theoretical answer (which is $\mu_x + \rho\frac{\sigma_1}{\sigma_2}(y - \mu_y)$ see e.g. this link under Bivariate conditional expectation).

b) Use the methods in Longstaff and Schwartz to compute $E(X|Y)$ with the Laguerre basis functions (first three) as described in page 122. Compare with the theoretical answer and the answer in part a)

c) (Optional) If your answer in a) or b) are not close enough, either increase the number of basis functions or the sample size to obtain satisfactory accuracy.

3. Consider the Black-Scholes model:

$$dS_t = rS_t dt + \sigma S_t d\widetilde{W}_t$$

where

$$S_0 = 50, \sigma = 0.2, r = 0.02.$$

We investigate the American put option with strike $K = 55, T = 2$ in this problem.

a) Use the method in Longstaff and Schwartz to find V_0 of this option.

b) Plot the sample density (or a histogram plot) of $\tau^* := \inf\{t \geq 0, V_t = (K - S_t)^+\}$ the optimal exercise time after 0.

c) Verify that $V_0 = \tilde{E}(e^{-r\tau^*}(K - S_{\tau^*})^+)$ numerically (using your simulation result above).

d) Plot the graph of $\tilde{E}(e^{-rt}V_t)$. Verify that it is a non-increasing graph (indicating the super-martingale nature of $e^{-rt}V_t$).

4. Extra Credit (5 points in midterm 2, must be turned in before midterm 2) For all intents and purposes, the Binomial model does a good job of approximating the Black-Scholes model and with more flexibility (firms do use Binomial model for pricing their instruments, only with more sophisticated calibrations to market indicators). Re-perform all the steps in part 3 with the Binomial model and verify that you obtain similar answers to part 3. (You will need to calibrate the parameters of the Binomial model to match the given volatility σ. This calibration is given e.g. in John Hull).