Homework 10 (Due 4/27/2016)

Math 622

April 21, 2016

1. The Hull-White interest model is defined in section 6.5. Read this section. You will see that the Hull-White model is also an affine-yield model and that one can find a formula for $B(t,T)$ by the same pde method we used in class for the two-factor Vasicek model (see also Shreve, pages 411-413).

a) Do Exercise 6.3, Shreve.

b) For the Hull-White model, as treated in Example 6.5.1, we would like to derive a stochastic differential equation model for the zero-coupon bond price itself. Using the results of Example 6.5.1 on the Hull-White model, show that $d_t[D(t)B(t,T)] = -\sigma D(t)C(t,T)B(t,T)\,d\widetilde{W}(t)$ for $t \le T$.

(Apply Itô's rule; use equations (6.5.8) and (6.5.9).)

(c) Let $\widetilde{\mathbf{P}}^T$ be the risk-neutral measure when $B(t,T)$ is used as a numéraire; see section 9.4.3. Use the expression for $d_t[D(t)B(t,T)]$ obtained in part b) to construct a process $\widetilde{W}^T$ that is a Brownian motion under $\widetilde{\mathbf{P}}^T$. Let $dS(t) = R(t)S(t)\,dt + \gamma S(t)\,d\widetilde{W}(t)$ (γ is the volatility here since we have already used σ). Write a stochastic differential for the forward price, $\mathrm{For}_S(t,T)$, in terms of $d\widetilde{W}^T(t)$.

2. Shreve, Exercise 10.2

3. Shreve, Exercise 10.3

4. Shreve, Exercise 10.7