

Homework 6

Math 485

November 6, 2015

In all of the following questions, suppose S_t follow the Black-Scholes model under a risk neutral measure Q :

$$dS_t = rS_t dt + \sigma S_t dW_t.$$

1. Derive the Black-Scholes formula. Try to go as far as you can before consulting the notes or textbook.
2. Compute V_0 for the following Euro-style derivatives:
 - a. $V_T = (S_T^\beta - K)^+$, β a constant.
 - b. $V_T = 1$ if $S_T < K$, $V_T = 0$ otherwise.
 - c. $V_T = S_T^\beta$, β a constant.
 - d. $V_T = \log(S_T)$.
 - e. $V_T = 1$ if $K_1 < S_T < K_2$, $V_T = 0$ otherwise.
 - f. $V_T = (K - S_T^\beta)^+$, β a constant.
3. Let $\rho = \frac{\partial V_0}{\partial r}$ where $V_T = (S_T - K)^+$. Prove that

$$\rho = KTe^{-rT}N(d_2),$$

where

$$d_2 = \frac{(r - \frac{1}{2}\sigma^2)T - \log(K/S_0)}{\sigma\sqrt{T}}$$