1. (5 pts) If possible, write $\mathbf{u}$ as a linear combination of vectors in S

$$\mathbf{u} = \begin{bmatrix} -1 \\ 11 \end{bmatrix}, S = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}.$$

Answer:

We consider the system $\begin{cases} x & +2y & = -1 \\ 3x & -y & = 11. \end{cases}$

Multiply the 2nd equation by 2 and add to the first gives $7x = 21$ or $x = 3$. Substitute in the 1st equation gives $y = -2$.

2. (5pts) Is the following statement true or false. If false give a counterexample.

The coefficients in a linear combination can always be chosen to be positive scalars.

Answer: False. Counterexample:

Let $\mathbf{u} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and $S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.

Then the only scalar so that $\mathbf{u}$ is in the linear combination of S is -1. If you're bothered with the fact that S has only 1 vector, we can let $S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ and the answer is basically still the same.

Note: I noticed some of you used Problem 1, in which $y = -2$ as a counterexample. I will accept this example, even though technically it is not entirely precise, using the material we have covered up to now. The reason is it might be possible that we can come up with other choices of x, y in Problem 1 that are positive (Which as you may supect, is impossible. That has to do with the fact that the system in Problem 1 has only 1 solution. But the criterion of a system to have only 1 solution is not covered yet in class).