

Math 485

NOTES ON FINANCIAL ASSETS AND MARKETS

A. EQUITIES

Basic Equities: Cash, foreign currency, stocks, bonds, money market or bank accounts.

- Basic equities have value in and of themselves.
- They are traded in well-regulated, transparent markets— e.g. stock exchanges.

Basic equities should be compared to **(Physical) Commodities:** These are: **1.** Physical goods, typically raw or partially processed agricultural or mining products, that are commercially traded; **2.** more generally, any economic goods.

B. RISK

The primary financial fact of life is **RISK!**

(Loose) Definition. In this course:

risk \approx exposure to **adverse** financial changes

but often, more generally:

risk \approx uncertainty of future prices, values;

Examples:

- stocks are risky assets;
- credit risk: risk that creditors default;
- foreign exchange risk; risk of currency exchange rate fluctuation to cash flows in foreign currency.

Hedging: Acting to reduce or to protect against risk.

C. INTEREST

Notation:

- Initial deposit is denoted by V_0 .
- Accumulated value at time t by V_t , t measured in years, (assuming no withdrawals).

Compounding.

Formula for the return on interest at nominal annual rate r compounded n times per year:

$$V_t = V_0 \left(1 + \frac{r}{n}\right)^{nt}$$

Formula for the return on continuously compounded interest at nominal annual rate r :

$$V_t = V_0 e^{rt} \quad \left(= \lim_{n \rightarrow \infty} V_0 \left(1 + \frac{r}{n}\right)^{nt}\right)$$

Present value: Let r = interest rate. Let today be $t = 0$. Let $I(t)$ denote 'income' due to us at time t ; $I(t) > 0$ means we receive $I(t)$; $I(t) < 0$ means we pay out $-I(t)$. The **present value** of $I(t)$ is its value to us today and this is

$$e^{-rt} I(t).$$

e^{-rt} is called the discount factor.

Generalizing, consider the income stream $I = (I(t_1), I(t_2), \dots, I(t_n))$ of payments at times $0 \leq t_1 < t_2 < \dots < t_n$. Its present value is defined to be

$$PV \triangleq \sum_1^n e^{-rt_i} I(t_i).$$

Present value can be used to compare the relative values of different income streams.

D. Leverage

Leverage is a financial arrangement which has a multiplying effect on the profit (or loss) on an investment relative to change in value of the invested assets.

We will see examples later.

E. FINANCIAL DERIVATIVES

Definition: A **financial derivative** (**derivative security, contingent claim**) is a contract between two parties for a future transaction exchanging cash and/or assets such that:

–the value of the transaction, the gain or loss to either party, depends on the value of other underlying variables at the transaction time.

The underlying variables and how the transactions depend on them are explicitly spelled out in the contract.

Example 1: Forward Contracts

Consider a basic equity or commodity with price, as a function of time, S_t . The equity might be a stock or foreign currency. The commodity might be raw material a firm needs to buy in the future.

In a forward contract on this equity, one party, say A (Alice), agrees to buy the equity from party B (Bob) **at a future time**. No money changes hands at the time A and B enter the contract. The forward contract fixes:

- The amount of asset A will purchase.
- The date T on which the purchase will take place; T is called the **delivery date**.
- The price per unit asset X , called the **delivery price**, of the purchase.

The profit/unit asset realized by A from the contract is called the **payoff** to A . It is

$$\text{payoff}/(\text{unit asset}) \text{ to } A = S_T - X,$$

because A pays X at time T for what is worth S_T .

Example 2: European Call Option on a Stock

Let S_t be the price process of a share of XYZ Corp.

In a European call option on XYZ stock, the **option holder** (Alice) has the right, but not the obligation, to purchase an agreed-upon number of XYZ shares from the option writer (Bob) at a specified time T for a specified price X per share. Alice will pay Bob some **premium** c for this right at the time the contract is entered.

T is the **expiration date**.

X is called the **strike price**.

What will happen? Alice (A) will **exercise the option**—call on Bob to sell at price X —if $S_T > X$. In this case, Alice could turn around and immediately sell for S_T what she bought for X , making a profit of $S_t - X$. But if $S_T \leq X$, Alice can acquire the stock more cheaply in the market and will let the option expire unexercised.

Therefore, the **payoff to A at expiration is:**

$$\text{payoff}/\text{share to } A = \max\{S_T - X, 0\}.$$

The **total payoff (net gain) to A is, ignoring discounting:**

$$\max\{S_T - X, 0\} - c$$

where c is the premium that Alice paid for the option. (This is the standard way to write the net payoff, but if one were accounting for discounting and wanted the total net payoff **at** T , one should use instead $\max\{S_T - X, 0\} - ce^{rT}$, where r is the prevailing interest rate for short term loans; but no one seems to use this corrected formula.)

In these examples one sees explicitly how the contract payoff depends upon the unknown future value of an underlying asset. The reason for the name *derivative* or *contingent claim* is precisely that the payoff “derives from” or is “contingent upon” other asset prices.

F. SOME TERMINOLOGY

- **Option.** Derivative contract in which one party holds the right to exercise or not.
- **Long position.** The party in the contract that is on the purchasing side of the contracted transaction. In options contracts, the party holding the exercise right has the long position. In forward or futures contracts, the party purchasing and taking delivery is long.
- **Short position.** Counterparty to the long position.
- **Option holder, owner or buyer.** The party in the long position.
- **Option writer.** Party in the short position.
- **Premium.** Price option holder pays to option writer.
- **Expiration date.** Final date at which an option can be exercised.
- **Payoff function.** The profit or loss, as a function of the underlying variables, of the contracted transaction to a party to the contract. In general discussions payoff will mean payoff to the long position.
- **Payoff diagram.** Graph of the payoff function.
- **Equity option.** Option written on an equity (stock) underlying.
- **Index option.** Option written on an index underlying.
- **Open Interest:** The open interest at a particular time for a specified option is the number of listed contracts of that option type held at that time.

G. Option types and classification

Options separate broadly into **calls** and **puts**.

- A **call** option gives the holder the option to buy an underlying.
- A **put** option gives the holder the option to sell an underlying.

Options are also classified into two broad types, **vanilla options** and **exotic options**.

- Vanilla options are common, widely available options, often exchange traded, with simple payoff functions based on a single underlying.
- Exotic options are those other than vanilla. They generally depend on underlyings in complicated and, indeed, exotic, ways and are traded over-the-counter.

Some Basic Vanilla Options

- Forward contracts;
- Futures; Futures are essentially forward contracts that are traded. We defer discussion.
- **European calls and puts:** These are call and put options on a single underlying. The designation *European* means the holder may exercise the option only at the expiration date T .
 - (a) The *European call* at strike X , expiration T , gives the holder the right to **buy** at time T for price X . The payoff to the long position is

$$\max\{S_T - X, 0\}.$$

- (b) The *European put* at strike X , expiration T , gives the holder the right to **sell** at time T for price X . The long payoff is

$$\max\{X - S_T, 0\}.$$

- **American calls and puts:** These are again options on a single underlying. The designation *American* means that the holder may exercise the option at any time up to and including the expiration date T . Most exchange traded options are American.

- (a) The *American call* at strike X , expiration T , gives the holder the right to **buy** at any time T^* , $T^* \leq T$, for price X . The payoff to the long position is

$$\max\{S_{T^*} - X, 0\}.$$

- (b) The *American put* at strike X , expiration T , gives the holder the right to **sell** at any time T^* , $T^* \leq T$ for price X . The long payoff is

$$\max\{X - S_{T^*}, 0\}.$$

Note: There is no significance to the appellations *European* or *American*. They could have been called *blue* and *green*.

Exotic options: There are a bewildering variety. Many are **path-dependent**, that is their payoff depends on the whole history of the underlying price up to the time of exercise, not just its price at the time of exercise. We give one example.

Example: **Asian call.** In the Asian call at strike X with expiration T , the holder has the right to ask the option writer for the difference between the average asset price up to time T and the strike. Thus the payoff to the option holder is

$$\max\left\{\frac{1}{T}\int_0^T S_t dt - K, 0\right\}.$$

H. Option markets

- Options are traded on **option exchanges** or **over-the-counter**.
- Option exchanges trade in standardized vanilla options, almost exclusively American calls and puts on stocks and stock indices. Standardization means that the exchange sets the contract sizes (shares of underlying per option contract), expiration dates, and strike prices. Traders need only negotiate number of contracts and price. A clearing house acts as the intermediary for all exchange transactions—purchasing, selling, exercising.
- One can close out a position on an option exchange at any time.

Why trade derivatives?

Three reasons are generally given: hedging, speculation, and arbitrage. We will discuss arbitrage later. Investors and financial officers can use derivatives to reduce the risk of transactions they need or want to make in the future. Of course, they pay a price for this either in terms of options premia, or reduced potential gain. Speculators can use derivatives to speculate on future price movements of an asset without investing in that asset itself. Hence derivatives can be used as leverage instruments.