

Math 485 - Some examples of financial derivatives

Sample Problems

1. Alice is short a European put on XYZ stock at strike \$30. The selling price is \$4. Under what circumstance does Alice gain?

The net payoff of a put to the short position is

$$c - \max\{X - S_T, 0\}$$

where X is the strike, c the premium and S_T the asset price at expiration. For the data of the problem the payoff per stock is thus $4 - \max\{30 - S_T, 0\}$, and this is positive as long as $S_T > \$26$.

2. Bob will receive £1,000,000 in three months. What type of derivative can Bob use to hedge against foreign exchange risk if Bob wants to sell his pounds for dollars when he receives them?

Let $S_t = \text{dollars/pound}$ at time t . Bob is at a disadvantage if S_t declines. He can hedge this by buying a put for selling pounds at some fixed strike price.

Or he could enter the short side of a forward contract on pounds at a fixed exchange rate.

3. In a **range forward** contract with delivery date T and delivery price thresholds $K_1 < K_2$, the holder agrees to buy at price K_1 if $S_T \leq K_1$, to buy at price S_T if $K_1 < S_T < K_2$, and to buy at price K_2 if $S_T > K_2$. Show that the holder's position is equivalent to being short a put at strike K_1 and long a call at strike K_2 .

This is easiest to see by drawing the diagram for the payoff at expiration of the range forward and of the portfolio in the options. The payoff at expiration to someone short a put at K_1 and long a call at K_2 is $\max\{S_t - K_2, 0\} - \max\{K_1 - S_T, 0\}$ and a case by case analysis shows

$$\max\{S_t - K_2, 0\} - \max\{K_1 - S_T, 0\} = \begin{cases} S_T - K_1, & \text{if } S_T < K_1; \\ 0, & \text{if } K_1 \leq S_T \leq K_2; \\ S_T - K_2, & \text{if } S_T > K_2. \end{cases}$$

4. A **bull spread** is the position: long one call at strike K_1 , short one call at strike K_2 , where $K_2 > K_1$. Analyze the payoff at expiration and explain the name.

The payoff function at expiration is

$$\max\{S_T - K_1, 0\} - \max\{S_T - K_2, 0\} = \begin{cases} 0, & \text{if } S_T < K_1; \\ S_T - K_1, & \text{if } K_1 \leq S_T \leq K_2; \\ K_2 - K_1, & \text{if } S_T > K_2. \end{cases}$$

An investor bullish about the underlying asset might be interested in this spread if he thinks the price will rise above K_1 . Of course, such an investor could also just purchase a call at K_1 . But if he thinks that it won't rise above level K_2 , the bull spread is better because it will cost less, since the investor will spend money to buy the call at K_1 , but receive money selling the call at K_2 .