Math 300 Intro Math Reasoning
Worksheet 05: Set Theory

(1) Prove that for every three sets A, B, C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(2) Prove that $\mathbb{Z} = \{r \in \mathbb{R} \mid \mathbb{Z} \cap (r, r+1) = \emptyset\}$

[Hint: you may use the fact that for any $z \in \mathbb{Z}$ there are no integers between z and $z+1$. You may also use "round up" and "round down".]

We define for every $A \subseteq \mathbb{R}$ and $r \in \mathbb{R}$

$$A + r = \{a + r \mid a \in A\}$$

(3) Compute (not proof):

  (1) $\{1, 5\} + 0.5$.
  (2) $\mathbb{N} + 1$.
  (3) $\mathbb{Z} + 1$.
  (4) $\emptyset + r$.

(4) Prove or disprove:

  (1) If $A \subseteq B$ then $A + r \subseteq B + r$.
  (2) If for some $r, s \in \mathbb{R}$, $A + r \subseteq B + s$ then $A \subseteq B$.
  (3) $A + 0 = A$

(5) Prove that for every $r \in \mathbb{R}$, $\mathbb{Q} + r = \mathbb{Q}$ if and only if $r \in \mathbb{Q}$.