

**Math 300 Intro Math Reasoning
Worksheet 04: Mathematical logic**

(1) Prove or disprove the following statement.

$$\forall x(\forall y((x < y) \Rightarrow (\exists z(x < z \wedge z < y))))).$$

(2) Prove that

$$\lim_{n \rightarrow \infty} \frac{n+1}{n} = 1$$

[Recall: $\lim_{n \rightarrow \infty} a_n = L$ means $\forall \epsilon > 0 \exists N \in \mathbb{N} \forall n \geq N |a_n - L| < \epsilon$.]

(3)

(a.) Prove that $1 + \sqrt{2}$ is irrational.

(b.) Prove that $\sqrt{3}$ is irrational.

(4) $A = \{1, 2, 3\}$, $B = \{1, 1, 2, 3\}$, $C = \{n \in \mathbb{N} \mid \exists y \in \mathbb{R}(|y| + |3 - n| \leq 3)\}$,
 $D = \{\{1\}, \{1, 2\}, \{1, 2, 3\}\}$, $E = \{1, \{1, 2, 3\}, 3\}$ $F = \{2^n - m \mid n \in \mathbb{N}, m \in \{0, 1\}\}$

(1) How many elements are in each of the sets?

(2) Determine if

(a) $A = B$.

(b) $A \subseteq E$.

(c) $A \in E$.

(d) $A = C$.

(e) $A \subseteq C$

(f) $E \subseteq D$.

(g) $A \subseteq F$

(h) $C \subseteq F$?