

Math 300 Intro Math Reasoning
Worksheet 03: Mathematical logic

(1) Prove the following statement: An integer is divisible by 5 if and only if its last digit is divisible by 5.

[Hint: To formally refer to the unit number of an integer n , decompose $n = 10k + d$ where k is some integer and $0 \leq d \leq 9$. Then d is the unit digit of n .]

(2) Let n and m be integers prove that if n is multiple of 6 or m is multiple of 9 then n^2m is a multiple of 9.

(3) Let a, b be integers with $b \neq 0$. Prove that any integer solution to the quadratic equation $x^2 + ax + b = 0$ divides b .

(4) Let x, y be real numbers. Prove that if $0 < x < y$ then there is $\epsilon > 0$ such that $\sqrt{x^2 + \epsilon} < y$.