

**Math 300 Intro Math Reasoning**  
**Worksheet 02: Mathematical logic**

(1) Consider the statement:

$\alpha =$  "Every real solution of  $x^2 + x - 6 = 0$  is positive."

(1) Formalize it using the propositional calculus.

(2) Give examples of sets of discourse  $A, B$  such that  $\alpha$  is true in  $A$  and  $\alpha$  is false in  $B$ .

(2) Write the negation of the following sentence **without** the negation symbol " $\neg$ " and determine whether it is true or false in the set  $\mathbb{R}$ :

" $(\exists x(x > 5)) \Rightarrow (\forall y(y > -100))$ ."

(3) What are all the  $x \in \mathbb{N}$  such that  $\exists y, x + y = 4$ ?

(4) Show that the following are not logically equivalent:

$$\forall x, \exists y, P(x, y) \text{ and } \exists y, \forall x, P(x, y)$$

Hint: Find a domain and interpretation for  $P(x, y)$  under which one of the formulas is true and the other is false.