Math 300 Intro Math Reasoning
Worksheet 01: Mathematical logic-Sols

(1) Prove that $P \implies Q$ and $\neg Q \implies \neg P$ are logically equivalent but that $P \implies Q$ and $Q \implies P$ are not logically equivalent.

Solutions:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\sim P$	$\sim Q$	$\sim Q \Rightarrow \sim P$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

(2) Prove that $\neg(P \wedge Q)$ and $(P \wedge \neg Q) \vee \neg P$ are logically equivalent.

(3) Prove that $P \iff Q \equiv (P \implies Q) \wedge (Q \implies P)$.

(4) Suppose that $\alpha \equiv T$ and $\beta \equiv F$, for each of the following determine if weather they are a tautology or a contradiction:

(1) $(\beta \wedge \alpha) \Rightarrow \beta$.
Solution: We claim that $(\beta \wedge \alpha) \Rightarrow \beta$ is a tautology. Suppose that v is a true value assignment, then $v(\beta) = F$ and also $v(\alpha \wedge \beta) = F$. Hence $v((\alpha \wedge \beta) \Rightarrow \beta) = T$.
(2) $\beta \wedge (\alpha \Rightarrow \beta)$.

(5) Decide whether the conclusion follows from the premises:

- Pre. 1: $A \Rightarrow (B \Rightarrow C)$
- Pre. 2: $\neg B \vee (\neg C)$
- Conclusion $\neg B \vee \neg A$.

Solution: It does follow. Suppose otherwise, that

$$(I) V(\sim B \vee \sim A) = F$$

but

$$(II) V(A \Rightarrow (B \Rightarrow C)) = T$$
$$(III) V(\neg B \vee (\neg C)) = T.$$

Then by (I) $V(A) = V(B) = T$ and by (III), $V(C) = F$. Thus $V(B \Rightarrow C) = F$ and $V(A \Rightarrow (B \Rightarrow C)) = F$, contradicting (II).