Intro to advanced mathematics-Midterm 2

MATH 215 (Instructor: Tom Benhamou) November 7, 2022

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this you will have 45 minutes during class. No material is allowed during the exam. The answers to the problems should be answered in the designated areas.

Problems

Problem 1. Answer the following items:

a. $\emptyset \in P(\emptyset) \times P(\emptyset)$. True \ <u>False</u>

b. For any sets A, B, C, D:
$(A \cap B) \setminus (C \cap D) = (A \setminus C) \cap (B \setminus D)$. True \ <u>False</u>

c. Compute $gcd(154, 278)$. solution:____2____

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Problem 2. Prove by induction that for all $n \in \mathbb{N}$, $7^n - 2^n$ is divisible by 5.

Solution:

- Base: For $n = 0$, $7^0 - 2^0 = 1 - 1 = 0$ is divisible by 5.
- Hypothesis: Suppose that $7^n - 2^n$ is divisible by 5.
- Step: We want to prove that $7^{n+1} - 2^{n+1}$ is divisible by 5. Indeed,

  $$7^{n+1} - 2^{n+1} = 7 \cdot 7^n - 2 \cdot 2^n = 7 \cdot 7^n - 7 \cdot 7^2 + 5 \cdot 2^n = 7(7^n - 2^n) + 5 \cdot 2^n$$

  By the induction hypothesis $7^n - 2^n$ is divisible by 5 and clearly $5 \cdot 2^n$ is divisible by 5. Hence $7^{n+1} - 2^{n+1}$ is divisible by 5. $\square$

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Problem 3. Prove that for any sets A, B, C:

$$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$$

Solution: We want to prove a set equality, let us prove it by a double inclusion:

- $\underline{(A \cap B) \setminus C \subseteq (A \setminus C) \cap (B \setminus C)}$: Let $a \in (A \cap B) \setminus C$. We want to prove that $a \in (A \setminus C) \cap (B \setminus C)$. By definition of difference $a \in A \cap B$ and $a \notin C$. By definition of intersection $a \in A$ and $a \in B$. Since $a \in A$ and $a \notin C$, $a \in A \setminus C$. Since $a \in B$ and $a \notin C$, $a \in B \setminus C$. Thus by definition of intersection $a \in (A \setminus C) \cap (B \setminus C)$.

- $\underline{(A \cap B) \setminus C \supseteq (A \setminus C) \cap (B \setminus C)}$: Let $a \in (A \setminus C) \cap (B \setminus C)$ we want to prove that $a \in (A \cap B) \setminus C$. By definition of intersection, $a \in A \setminus C$ and $a \in B \setminus C$. By definition of difference $a \in A$ and $a \notin C$ and also $a \in B$ and $a \notin C$. Since $a \in A$ and $a \in B$, it follows that $a \in A \cap B$. Since $a \notin C$, it follows that $a \in (A \cap B) \setminus C$, as wanted. $\square$