Introductory Linear Algebra-Midterm Example

MATH 250   (Instructor: Tom Benhamou)   September 19, 2024

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this you will have one hour. The identities file will be appended to the exam and no other material is allowed. The answers to the problems should be answered in the designated areas.

Problems

Problem 1. For each of the following statements determine if it is true are false. Provide a counter example if false. No explanation is required if true (circle the correct answer):

    a. 3 vectors in $\mathbb{R}^5$ are linearly independent.   True \ False

        counter example: _____

    b. 7 vectors in $\mathbb{R}^6$ are linearly dependent.   True \ False

        counter example: _____

    c. the columns of a 5×5 matrix with two equal rows are linearly dependent.   True \ False

        counter example: _____

Problem 2. Determine whether the sequence $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$ is linearly independent and/or spanning.

Solution:

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Problem 3. Consider the following linear system in the variables x, y, z ($\alpha \in \mathbb{R}$ is a parameter):

$$\begin{cases} x + y + z = 1 \\ x + y + \frac{\alpha}{2}z = 2 \\ x + y + \alpha z = 1 \end{cases}$$

Determine for which α there is no solution, for which α there is a unique solution and for which α there are ∞-many solutions. Write the general solution in parametric representation for any α such that there is a solution

   solution: