MidTerm II- Set Theory fall 2023

MATH 361 (Instructor: Tom Benhamou) Nov 17

Instructions

The midterm duration is 1 hour and 20 min, and consists of 4 problems, each worth 26 points (The maximal grade is 100). The answers to the problems should be written in the designated areas.

Problems

Problem 1. Let us define recursively $A_0 = \emptyset$ and $A_{n+1} = P(A_n)$. Prove by induction that for every n, $A_n \subseteq A_{n+1}$

Solution:

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Problem 2. Give the definition of a linear ordering.

(a) Give the definition of an isomorphism between two linear orders $\langle L_1, < \rangle$ and $\langle L_2, \prec \rangle$.

Prove or disprove each of the following statements:

(i) $\langle \mathbb{Q} \setminus \mathbb{Z}, < \rangle \simeq \langle \mathbb{Q} \setminus \mathbb{N}, < \rangle$.

(ii) $\langle \mathbb{R}, < \rangle \simeq \langle \mathbb{R} \setminus (0, 1), < \rangle$.

Solution:

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Problem 3. Fix a natural number $N > 0$. A function $f : \mathbb{N} \to \{0,1\}$ is called N-periodic if for every $n \in \mathbb{N}$, $f(n+N) = f(n)$. For any $N > 0$, let A_N be the set of all N-periodic functions. Show that

$$A_N \approx \{0,1\}^N = \{0,1\} \times \{0,1\} \times \ldots \times \{0,1\}$$

[Instructions: Half of the points are given for a correct definition of a bijection, the other half is the proof that the defined function is indeed a bijection.]

Solution

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Problem 4. A function f is called periodic if there is $N \in \mathbb{N}_+$ such that f is N-periodic. Show that the set A of all N-periodic functions is infinitely countable. [Remark: You can use Problem 3 even if you did not prove it.]

Solution