Homework 1

MATH 361 (due September 22) September 15, 2022

Problem 1. Prove or disprove the following items:

(a) $\{1, -1\} \subseteq \mathbb{Z}$.

(b) $7 \in \{n \in \mathbb{N} \mid |n^2 - n - 3| \leq 5\}$.

(c) $27 \in \{n^2 - n - 3 \mid n \in \mathbb{N}\}$.

(d) $-3 \in \{n^2 - 3 \mid n \in \mathbb{N}_+\}$.

(e) $\{1, -1\} \in \{X \subseteq \mathbb{Z} \mid 2 \in X\}$.

(f) $\{r \in \mathbb{R} \mid \exists q \in \mathbb{Q}. r + q \in \mathbb{Q}\} = \mathbb{Q}$.

(g) $\{-1, 0, 1\} \subseteq \{x \in \mathbb{N} \mid x^2 = |x|\}$. (Here $|x|$ is the absolute value of the real number x)

(h) $\{x \in \mathbb{R} \mid \{x, x + 1\} \subseteq [0, 2)\} \subseteq [0, 1]$.

(i) $\mathbb{Q} \subseteq \{x \in \mathbb{R} \mid |\{x, x + \sqrt{2}\} \cap \mathbb{Q}| = 1\}$

Problem 2. Prove that if A, B, C are sets then

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Problem 3. Let $\mathcal{B}$ be a nonempty set of sets and let A be any set. Show that

(a) $A \cap \bigcup \mathcal{B} = \bigcup \{A \cap B \mid B \in \mathcal{B}\}$.

(b) $A \setminus \bigcap \mathcal{B} = \bigcup \{A \setminus B \mid B \in \mathcal{B}\}$.

Problem 4. Let A, B be sets. prove that for any $a \in A$ and $b \in B$, $\langle a, b\rangle \in P(P(A \cup B))$. Conclude formally from the axioms that there is a unique set D which equals $A \times B$. Namely, prove that:

(a) There is a set D with the property that for every x, $x \in D$ if and only if $x = \langle a, b\rangle$ for some $a \in A$ and $b \in B$.

(b) If D, D' both have the property described in (a) then $D = D'$.

Problem 5. Prove that for every sets A, B, C,

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

Additional problems:

Problem 6. Prove implications (3) $\Rightarrow$ (4) and (4) $\Rightarrow$ (1) of Proposition 2.9.

Problem 7. Let X and Y be sets.

(i) Prove that $Y \setminus (Y \setminus X) = X \cap Y$.

(ii) Prove that $X \subseteq Y$ if and only if $X \cup Y = Y$.

(iii) Deduce that $X \subseteq Y$ if and only if $Y \setminus (Y \setminus X) = X$.

Problem 8. Compute the following sets. No proof required.

1. $\Big\{a + b : a \in \{0, 5\}, b \in \{2, 4\}\Big\} \setminus \{7, 10\}$.
2. $(1, 3) \cup [2, 4)$
3. $\mathbb{Z} \cap [0, \infty)$

4. $\mathbb{N}_{even} \Delta \mathbb{N}_+$

Problem 9. Prove that for every two sets A, B the following are equivalent:

- $A \subseteq B$.
- $P(A \cup B) = P(B)$.
- $P(A) \subseteq P(B)$.

[Remember: You are allowed to use the propositions and statements which appear in the class notes.]