Homework 4

MATH 361 (due October 20) October 13, 2023

Problem 1. Prove that rational addition defined by:

$$[\langle n, m\rangle]_{\sim_Q} + [\langle n', m'\rangle]_{\sim_Q} = [\langle nm' + n'm, mm'\rangle]_{\sim_Q}$$

does not depend on the choice of representatives.

Problem 2. For two function $f, g \in {}^{\mathbb{N}}\mathbb{N}$ deinfe

$$f \leq^* g \iff \exists N \forall n \geq N,\ f(n) \leq g(n)$$

1. Prove that $\leq^*$ is not anti-symmetric.
2. Let
$$E = \{\langle f, g\rangle \in ({}^{\mathbb{N}}\mathbb{N})^2 \mid \exists N \forall n \geq N, f(n) = g(n)\}$$
Prove that E is an equivalence relation.
3. Prove that the relation $[f]_E \leq^* [g]_E$ iff $f \leq^* g$ does not depend on the choice of representatives and partially orders ${}^{\mathbb{N}}\mathbb{N}/E$.

Problem 3. Prove or disprove $\langle \mathbb{N}, <\rangle \simeq \langle \mathbb{N} \times \mathbb{N}, <_{Lex}\rangle$

Problem 4. Prove that for all $m \in \mathbb{N}$, either $m = \emptyset$ or $\emptyset \in m$. [Hint: Show that $S = \{m \in \mathbb{N} \mid m = \emptyset \text{ or } \emptyset \in m\}$ is inductive.]

Problem 5. Given distributively in the natural numbers, prove that the multiplication is associative

Problem 6. Prove that $(n \cdot m)^k = n^k \cdot m^k$.