

**Math 300 Intro Math Reasoning**  
**Worksheet 07: Functions**

(1)

- (1)  $f_1 : \mathbb{R} \rightarrow \text{codom}(f_1)$ , defined by  $f_1(x) = 5x - x^2$ .  
Compute  $f_1(1)$ .
- (2)  $f_2 : P(\mathbb{R}) \rightarrow \text{codom}(f_2)$ , defined by  $f_2(x) = x \cap \mathbb{N}$ .  
Compute  $f_2(\{1, \pi, -1\})$  and  $f_2((-\infty, 5))$ .
- (3)  $f_3 : P(\mathbb{R}) \rightarrow \text{codom}(f_3)$ , defined by  $f_3(X) = \langle X \cap \mathbb{N}, X \cap \mathbb{Z}, X \cap \mathbb{Q} \rangle$ .  
Compute  $f_3(\mathbb{Z})$  and  $f_3([-1, 1])$ .

**Solution.**

- 1)  $f_1(1) = 5(1) - 1^2 = 4$
- 2)  $f_2(\{1, \pi, -1\}) = \{1\}$   
 $f_2((-\infty, 5)) = [0, 5)$
- 3)  $f_3(\mathbb{Z}) = \langle \mathbb{N}, \mathbb{Z}, \mathbb{Z} \rangle$   
 $f_3([-1, 1]) = \langle \{0, 1\}, \{-1, 0, 1\}, \{n \in \mathbb{Q} \mid -1 \leq n \leq 1\} \rangle$

(2) For each of the functions from the previous exercise, find their domain and codomain.

**Solution.**

- 1)  $\text{dom}(f_1) = \mathbb{R}, \text{codom}(f_1) = \mathbb{R}$
- 2)  $\text{dom}(f_2) = P(\mathbb{R}), \text{codom}(f_2) = P(\mathbb{N})$
- 3)  $\text{dom}(f_3) = P(\mathbb{R}), \text{codom}(f_3) = P(\mathbb{N}) \times P(\mathbb{Z}) \times P(\mathbb{Q})$

(3) Prove that for any two functions  $f : A \rightarrow B$  and  $g : B \rightarrow C$ , and any  $X \subseteq A$ ,  $(g \circ f) \upharpoonright X = g \circ (f \upharpoonright X)$ .

**Solution.** Let us prove function equality. Let  $x \in X$ , WTP  $(g \circ f) \upharpoonright X(x) = g \circ (f \upharpoonright X)(x)$ .  
Indeed,

$$(g \circ f) \upharpoonright X(x) = (g \circ f)(x) = g(f(x)) = g(f \upharpoonright X(x)) = g \circ (f \upharpoonright X)(x)$$