

Math 300 Intro Math Reasoning
Worksheet 06: Set Theory

(1) Prove by induction that

$$1 \cdot 2 + 3 \cdot 4 + \dots + (2n - 1) \cdot 2n = \frac{n(n + 1)(4n - 1)}{3}$$

(2) Prove that $A \subseteq B$ if and only if $P(A) \subseteq P(B)$.

(3) Define

$$t \cdot \langle \alpha_1, \dots, \alpha_n \rangle = \langle t \cdot \alpha_1, \dots, t \cdot \alpha_n \rangle$$

and denote by $\vec{0} = \langle 0, 0, \dots, 0 \rangle$. Prove that for every $t \in \mathbb{R}$ and $\vec{\alpha} \in \mathbb{R}^n$, if $t \cdot \vec{\alpha} = \vec{0}$, then either $t = 0$ or $\vec{\alpha} = \vec{0}$.