Math 300 Intro Math Reasoning
Worksheet 05: Set Theory

(1) Prove by induction that
$$1 \cdot 2 + 3 \cdot 4 + \ldots + (2n-1) \cdot 2n = \frac{n(n+1)(4n-1)}{3}$$

(2) Prove that $A \subseteq B$ if and only if $P(A) \subseteq P(B)$.

(3) Define
$$t \cdot \langle \alpha_1, \ldots, \alpha_n \rangle = \langle t \cdot \alpha_1, \ldots, t \cdot \alpha_n \rangle$$
and denote by $\vec{0} = \langle 0, 0, \ldots, 0 \rangle$. Prove that for every $t \in \mathbb{R}$ and $\vec{\alpha} \in \mathbb{R}^n$, if $t \cdot \vec{\alpha} = \vec{0}$, then either $t = 0$ or $\vec{\alpha} = \vec{0}$.