Math 300 Intro Math Reasoning
Worksheet 02: Mathematical logic

(1) Consider the statement:

$\alpha =$"Every real solution of $x^2 + x - 6 = 0$ is positive."

(1) Formalize it using the predicate calculus. **Solution** $\forall x \in \mathbb{R}(x^2 + x - 6 = 0 \Rightarrow x > 0)$
(2) Give examples of sets of discourse A, B such that α is true in A and α is false in B.
Solution: If $A = [0, \infty)$, then α is true in A since we only range on non-negative x's and if $x^2 + x - 6 = 0$ then $x = 3 > 0$. If $B = \mathbb{R}$ then α is false since for example $x = -2$ is a solution to the equation which is negative.

(2) Write the negation of the following sentence **without** the negation symbol "¬" and determine whether it is true or false in the set $\mathbb{R}$:

"$(\exists x(x > 5)) \Rightarrow (\forall y(y > -100))$."

Solution: $\sim ((\exists x(x > 5)) \Rightarrow (\forall y(y > -100))) \equiv \exists x(x > 5) \wedge \exists y(y \leq -100)$. The negation is true.

(3) Compute $Tr^{\mathbb{N}}(\exists y,\ x + y = 4)$

Solution: $Tr^{\mathbb{N}}(\exists y(x + y = 4)) = \{0, 1, 2, 3, 4\}$.

(4) Prove that if a divides b then a divides $bc + ad$.

Solution: Suppose that a divides b. WTP $a|bc + ad$. By the assumption, there is $k \in \mathbb{Z}$ such that $b = ak$. Therefore letting $k' = kc + d$ we have

$$ak' = a(kc + d) = akc + ad = bc + ad$$

Since $k' \in \mathbb{Z}$, we conclude that a divides $bc + ad$.