Math 300 Intro Math Reasoning Worksheet 02: Mathematical logic

(1) Prove the following statement: An integer is divisible by 5 if and only if its last digit is divisible by 5.

[Hint: To formally refer to the unit number of an integer n, decompose n = 10k + d where k is some integer and $0 \le d \le 9$. Then d is the unit digit of n.]

(2) Let a, b be integers with $b \neq 0$. Prove that any integer solution to the quadratic equation $x^2 + ax + b = 0$ divides b.

(3) Compute the negation and prove or disprove the following statement.

 $\forall x (\forall y ((x < y) \Rightarrow (\exists z (x < z \land z < y)))).$