Math 300 Intro Math Reasoning
Worksheet 11: Systems of representatives

(1) Let $E = \{\langle X, Y \rangle \in P(\mathbb{R})^2 \mid 2023, 2024 \notin X \Delta Y\}$ be an equivalence relation over $P(\mathbb{R})$. Compute $|P(\mathbb{R})/E|$.

(2) Consider the equivalence relation over $\sim$ over $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ defined by $\langle z_1, z_2 \rangle \sim \langle z_3, z_4 \rangle$ if and only if $z_1 z_4 = z_2 z_3$. Define the "addition" of equivalence classes as

$$[\langle z_1, z_2 \rangle]_\sim + [\langle z_3, z_4 \rangle]_\sim = [\langle z_1 z_4 + z_2 z_3, z_2 z_4 \rangle]_\sim$$

Prove that this operation is well-defined and does not depend on the choice of representatives.