

Math 300 Intro Math Reasoning
Worksheet 10: Equivalence relations

(1) For each of the following relation check whether it is reflexive symmetric or transitive:

- (1) $\{ \langle a, b \rangle \in \mathbb{R}^2 \mid a + b = 350 \}$.
- (2) $\{ \langle a, b \rangle \in \mathbb{R}^2 \mid |a - b| < 1 \}$.
- (3) $\{ \langle X, Y \rangle \in P(\mathbb{R})^2 \mid 3 \notin X \Delta Y \}$.

(2) Let $E_n = \{ \langle z_1, z_2 \rangle \in \mathbb{Z}^2 \mid n \text{ divides } z_1 - z_2 \}$. For $n > 1$, Show that $z_1 E_n z_2$ if and only if $z_1 \pmod n = z_2 \pmod n$

[Hint: write $z_i = q_i n + r_i$ where $0 \leq r_i < n$ for $i = 1, 2$, and recall that if $n \mid m$ where $0 < n, m$ are natural numbers then $n \leq m$.]