Mathematical Reasoning (300)
2nd Midterm-Perparation

March 22, 2024

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this, you will have 1 hour during class. Your solutions should be written in the designated areas.

Practice problems

Problem 1. Prove that $(A \setminus B) \setminus C = (A \setminus C) \setminus (B \setminus C)$.

Problem 2. Prove by induction that for every $n \in \mathbb{N}_+$, $1+3+...+(2n-1) = n^2$.

Problem 3. Prove that for every integer $n > 0$, $n, n+1$ are coprime.

Problem 4. Prove that for all $n \in \mathbb{N}$, $9^n - 5^n$ is divisible by 4.

Problem 5. Express the following sets using the list principle. No proof required.

1. $(-5, 5) \cap \mathbb{Z}$.

2. $\{\emptyset, 1\} \times \{n \in \mathbb{N} \mid |P(\{1, ..., n\})| < 4\}$.

3. $\{\langle x, y \rangle \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \cap \{\langle x, x \rangle \mid x \in \mathbb{R}\}$.

Problem 6. Compute the following

1. Compute A_3, where A_n is defined recursively by $A_0 = \emptyset$ and $A_{n+1} = A_n \cup \{A_n\}$.

2. a_4 where a_n is defined recursively by $a_0 = 0$ and $a_{n+1} = 2^{a_n}$