Homework 9

MATH 300 (due Nov 12) Nov 15, 2024

Problem 1. Prove that if $A \sim B$ and $B \sim C$ then $A \sim C$.

Problem 2. Prove the following items:

1. $\mathbb{N} \setminus \{2023, 2024\} \sim \mathbb{N}_{even}$.

2. $P(\mathbb{N}) \setminus \{\emptyset\} \sim P(\mathbb{N})$.

3. $(0, 1) \sim (0, \infty)$.

4. $\mathbb{Z} \times [0, 1) \sim \mathbb{R}$.

Problem 3. Prove that for every $\alpha < \beta$ real numbers $(\alpha, \beta) \approx (0, 1)$. [Hint: First stretch/shrink $(0, 1)$ to have length $\beta - \alpha$, then shift it by $+c$ as we did in class.]

Problem 4. Show that ${}^{\mathbb{N}}\{0,1\} \times {}^{\mathbb{N}}\{0,1\} \approx {}^{\mathbb{N}}\{0,1\}$.

[Hint: Use the interleaving function $F : ({}^{\mathbb{N}}\{0,1\})^2 \to {}^{\mathbb{N}}\{0,1\}$ defined by

$$F(\langle f, g\rangle)(n) = \begin{cases} f(\frac{n}{2}) & n \in \mathbb{N}_{even} \\ g(\frac{n-1}{2}) & n \in \mathbb{N}_{odd} \end{cases}$$

as the witnessing bijection.]