

## Homework 8

MATH 300

(due April 5)

March 29, 2024

---

**Problem 1.** Prove that if  $f : A \rightarrow B$ ,  $g : B \rightarrow C$  are surjections then  $g \circ f$  is a surjection.

# Homework 8

MATH 300

(due April 5)

March 29, 2024

---

**Problem 2.** Prove or disprove the following items:

1. If  $f : A \rightarrow B$  is injective, then for every  $X \subseteq A$ ,  $f \upharpoonright X$  is injective.
2. If  $f : A \rightarrow B$  is surjective, then for every  $X \subseteq A$ ,  $f \upharpoonright X$  is surjective.

## Homework 8

MATH 300

(due April 5)

March 29, 2024

---

**Problem 3.** Prove that if  $f : A \rightarrow B$  is a function such that for some  $X \subseteq A$ ,  $f \upharpoonright X : X \rightarrow B$  is onto  $B$ , then  $f$  is not injective.

## Homework 8

MATH 300

(due April 5)

March 29, 2024

---

**Problem 4.** For each of the following functions, determine if it is injective/surjective and prove your answer.

1.  $f_1 : \mathbb{R} \rightarrow \mathbb{R}$ , defined by  $f_1(x) = 5x - x^2$ .

2.  $f_2 : \mathbb{R} \rightarrow P(\mathbb{R})$ , defined by  $f_2(x) = \{x^2\}$ .

3.  $f_3 : P(\mathbb{R}) \rightarrow P(\mathbb{N})$ , defined by  $f_3(x) = x \cap \mathbb{N}$ .

4.  $f_4 : P(\mathbb{N}) \rightarrow \mathbb{N}$ , defined by  $f_4(x) = \begin{cases} \min(x) & 4 \in x \\ 0 & \text{else} \end{cases}$ .

5.  $f_5 : P(\mathbb{R}) \rightarrow P(\mathbb{N}) \times P(\mathbb{Z}) \times P(\mathbb{Q})$ , defined by

$$f_5(X) = \langle X \cap \mathbb{N}, X \cap \mathbb{Z}, X \cap \mathbb{Q} \rangle$$

6.  $f_6 : \mathbb{N} \times \mathbb{Z} \rightarrow P(\mathbb{N})$ , defined by  $f_6(\langle n, m \rangle) = \{x \in \mathbb{N} \mid n < x < m\}$ .

## Homework 8

MATH 300

(due April 5)

March 29, 2024

---

**Problem 5.** In the following items, no proof required (just a formal definition of the functions):

1. Find an injective function  $f : \mathbb{N} \rightarrow P(\mathbb{N})$ .
2. Find a surjective function  $f : \mathbb{Z}^2 \rightarrow \mathbb{Q}$ .
3. (\*Optional) Find an injective function  $f : \mathbb{R} \rightarrow P(\mathbb{Q})$  [Hint: Use the density of the rationals inside the reals].
4. Find a surjective function  $f : \mathbb{N} \rightarrow \mathbb{Z}$ .