Homework 6

MATH 300 | (due Oct 18) | Oct 11, 2024

Problem 1. Describe the set $P(\{\emptyset, \{\emptyset\}\})$ using the list principle. No proof required.

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Problem 2. Prove that for every two sets A, B the following are equivalent:

- $A \subseteq B$.
- $P(A \cup B) = P(B)$.
- $P(A) \subseteq P(B)$.

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Problem 3. Prove that for every natural number n, the number $4^n - 1$ is multiple of 3.

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Problem 4. Prove that for every natural number n, the number $17n^3 + 103n$ is multiple of 6.

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Problem 5. Prove that for every natural number n,

$$1 + 4 + 4^2 + \cdots + 4^n = \frac{4^{n+1} - 1}{3}.$$

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Problem 6. Prove that for all odd natural numbers n, the number $2^n + 1$ is multiple of 3.

[Hint: First find an equivalent statement about all positive integers. Then use induction.]

Next problem is optional and will not be graded for points.

Problem 7. Criticize the following obviously wrong argument:

> «All horses are the same color. Specifically, every finite set of horses is monochromatic.»
>
> *Proof.* We argue by induction. The statement is clearly true for sets of size 1. Assume by induction that all sets of n horses are monochromatic, and consider a set of size $n + 1$. The first n horses are all the same color. The last n horses are all the same color. Because of the overlap, this means that all $n + 1$ horses are the same color. So by induction, all finite sets of horses are all the same color, and so all horses are the same color. □

Next problem is optional.

Problem 8. Suppose $x \geq -1$. Prove that for all natural number n,

$$(1 + x)^n \geq 1 + nx.$$

Next problem is optional.

Problem 9. Suppose $x \geq -1$. Prove that for all natural number n,

$$(1 + x)^n \geq 1 + nx.$$