

## Homework 3

MATH 300

(due Feb 16)

Feb 9, 2022

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**Problem 1.** Apply each of the following claims to two specific examples of your choice or find a counterexample. In your solution, you should provide the examples and what you have concluded from the statements:

- a. Suppose that  $n$  is a integer, such that 6 divides  $n(n + 1)(n + 2)$  then 24 divides  $n(n + 1)(n + 2)(n + 3)$ .
- b. Suppose that  $x, y, z$  are three integers such that  $x^2 + y^2 = z^2$ , then either 3 divides  $x$  or 3 divides  $y$ .

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**Problem 2.** Prove the following equivalences (using a double implication):

An integer is divisible by 4 if and only if its last two digits form a number divisible by 4.

[Hint: Decompose  $n = 100l + d$  where  $k, l$  is some integers and  $0 \leq d \leq 99$ . Then the number  $d$  is the last two digits.]

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**Problem 3.** Prove that if  $a$  and  $b$  are odd integers, then  $a^2 - b^2$  is a multiple of 8.

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**Problem 4.** Let  $a, b, c$  be integers. Prove that if  $a^2 + b^2 = c^2$ , then  $abc$  is even.