Homework 3-Sols

MATH 300 (due Sep 27) Sep 20, 2024

Problem 1. Apply each of the following claims (no need to prove them!) to two specific examples of your choice or find a counterexample. In your solution, you should provide the examples and what you have concluded from the statements:

a. Suppose that n is a integer, then 24 divides $n(n+1)(n+2)(n+3)$.

b. Suppose that x, y, z are three integers such that $x^2 + y^2 = z^2$, then either 3 divides x or 3 divides y.

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Problem 2. Prove the following equivalences (using a double implication):

An integer is divisible by 4 if and only if its last two digits form a number divisible by 4.

[Hint: Decompose $n = 100l + d$ where k, l is some integers and $0 \leq d \leq 99$. Then the number d is the last two digits.]

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Problem 3. Prove that if a and b are odd integers, then $a^2 - b^2$ is a multiple of 8.

Problem 4. Let a, b, c be integers. Prove that if $a^2 + b^2 = c^2$, then abc is even.

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Problem 5. (1) Prove that for every rational number $q \in \mathbb{Q}$, $\sqrt{2} \cdot q$ is irrational.

(2) Prove or disprove: the sum or irrational numbers is irrational.

(3) Prove that $\sqrt{5}$ is irrational.

(4) (optional) Formulate a conjecture for the rationality and irrationality of real numbers of the form $\sqrt{n}$.