

**Problem 1.** Formalize each of the following statements using the predicate calculus.

(a) Every real solution of  $x^2 - 5x + 6 = 0$  is positive.

**Solution.**  $\forall x \in \mathbb{R}(x^2 - 5x + 6 = 0 \Rightarrow x > 0)$

(b) Every prime number is greater than 1.

**Solution.**  $\forall p \in \mathbb{N}((\forall n \in \mathbb{N}(n|p \Rightarrow n = 1 \vee n = p)) \Rightarrow p > 1)$

**Problem 2.** For each of the following statements, write the negation of the sentences **without** the negation symbol “ $\neg$ ”, and prove the negation:

1.  $\exists \epsilon((\epsilon > 0) \wedge (\forall x(x > 0 \Rightarrow x > \epsilon)))$ . **Solution.**

$$\exists \epsilon((\epsilon > 0) \wedge (\forall x(x > 0 \Rightarrow x > \epsilon))) \equiv \forall \epsilon((\epsilon \leq 0) \vee (\exists x((x > 0) \wedge (x \leq \epsilon)))$$

2.  $\forall x((x > 5) \Leftrightarrow (\forall y(y > -100)))$ .

(Hint: Recall that  $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$ )

**Solution.**

$$\begin{aligned} \forall x((x > 5) \Leftrightarrow (\forall y(y > -100))) &\equiv \\ &\equiv \exists x(((x > 5) \wedge (\exists y(y \leq -100))) \vee ((x \leq 5) \wedge (\forall y(y > -100)))) \end{aligned}$$

**Problem 3.** Prove the following statement:

If both  $a$  and  $b$  are divisible by  $n$ , then  $a - b$  is divisible by  $n$ .

**Solution.** Suppose that  $a$  and  $b$  are divisible by  $n$ . WTP  $a - b$  is divisible by  $n$ . By assumption, there are integers  $k, l$  such that  $a = kn$  and  $b = ln$ . Define  $t = k - l$ , then  $nt = n(k - l) = nk - nl = a - b$ . Hence  $a - b$  is divisible by  $n$ . Therefore if  $n$  divides  $a$  and  $b$  then  $n$  divides  $a - b$ .

**Problem 4.** Prove the following implication:

If  $n$  is even then  $n + 2$  is even.

**Solution.** Suppose that  $n$  is even. WTP  $n + 2$  is even. By assumption  $2|n$  and therefore there is  $k$  such that  $n = 2k$ . Define  $t = k + 1$ , it follows that  $2t = 2(k + 1) = 2k + 2 = n + 2$ . Hence  $n + 2$  is even. Therefore if  $n$  is even then  $n + 2$  is even.