Introductory Linear Algebra-Midterm III
Preparation questions

MATH 250 (Instructor: Tom Benhamou) November 18, 2024

Instruction

The structure and instructions for Midterm III is identical to Midterm I.

Problems

Problem 1. Anwer the following questions. No explanation is required:

a. rank($\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 8 & 1 \end{bmatrix}$) =________________________

b. There is a 6×4 matrix A with nullity$(A) = 2$ and rank$(A) = 2$

False \ True and give an example: ________________________

c. $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$. True \ False

d. $\{\begin{bmatrix} x \\ y+1 \\ z \,| \end{bmatrix} \mid x, y, z \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$. True \ False

e. For any bases $\mathcal{B}, \mathcal{C}$ of a vector space V, the matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ is invertible.

True \ False

counter example: ________________________

f. For any $m \times n$-matrix A of rank k, erasing the last column results in a matrix of rank $k-1$. True \ False

counter example: ______________________________

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Problem 2. Suppose that $S : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $S(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x - z \\ z - 2y - x \end{bmatrix}$.

Find bases for Ker(S) and Im(S).

Problem 3. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. For each eigenvalue of A present the eigenspace as the span of vectors.

Problem 4. Let $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be the map $T(p) = p + p'$, where p' is the derivative of p. Is T an isomorphism?

Problem 5. Let H be the subspace of $M_{3\times 3}(\mathbb{C})$ consisting of all matrices A with $A_{11} + A_{22} + A_{33} = 0$.

- Prove that H is a subspace of $M_{3\times 3}(\mathbb{C})$.
- What is dim(H)?

Problem 6. Let $\mathcal{B} = \{\begin{bmatrix} 1 \\ i \\ 1+i \end{bmatrix}, \begin{bmatrix} 1+i \\ 2 \\ -1-i \end{bmatrix}, \begin{bmatrix} 1+2i \\ 3-i \\ -i \end{bmatrix}\}$, show that $\mathcal{B}$ is a basis for $\mathbb{C}^3$ and compute $[\begin{bmatrix} 2-i \\ -1+3i \\ 1+2i \end{bmatrix}]_{\mathcal{B}}$.

Problem 7. Prove that if $\bar{x}$ is an eigenvector for A, then $\bar{x}$ is an eigenvector for A^2.

Problem 8. Suppose that $\mathcal{B} = \{\bar{b}_1, \bar{b}_2, \bar{b}_3, \bar{b}_4\}$ is a basis for V and $C = \{\bar{b}_1 + \bar{b}_1, \bar{b}_1 + \bar{b}_2, \bar{b}_1 + \bar{b}_3, \bar{b}_1 + \bar{b}_4\}$. Show that C is also a basis for V and compute $\underset{\mathcal{B}\leftarrow C}{P}$.

Problem 9. Let $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ and $\{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ be two LI sequences of vectors in V.

(a) Is $\{\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{u}_1, \bar{u}_2, \bar{u}_3\}$ LI?

(b) Show that if the only vector in $Sp(\{\bar{v}_1, \bar{v}_2, \bar{v}_3\})$ and in $Sp(\{\bar{u}_1, \bar{u}_2, \bar{u}_3\})$ is the zero vector then $\{\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{u}_1, \bar{u}_2, \bar{u}_3\}$ is LI.