Introductory Linear Algebra-Midterm II

MATH 250 (Instructor: Tom Benhamou) October 28, 2024

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this you will have one hour. No material is allowed. The solutions to the problems should be written in the designated areas and the "extra page" at the end. Detailed explanations for your solutions are required unless stated otherwise.

Full Name (PRINT): _____

Net ID: _____

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Problems

Problem 1. For each of the following statements determine if it is true are false. Provide a counterexample if false. No explanation is required if true (circle the correct answer):

a. If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation, then $\{T(\bar{e}_1), T(\bar{e}_2)\}$ are linearly independent. True \ False

counter example: _____

b. If A is an invertible martrix and $\alpha \neq 0$ is a scalar, then $\alpha \cdot A$ is invertible. True \ False

counter example: _____

c. Let A be an $(n+1)\times n$-matrix such that for every $\bar{b} \in \mathbb{R}^{n+1}$, $A \cdot \bar{x} = \bar{b}$ has a solution. Then erasing the last row from A results in an invertible matrix. True \ False

counter example: _____

d. If A, B are non square matrices then $A \cdot B$ is not invertable. True \ False

counter example: _____

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Problem 2. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is given by $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} -2y \\ 3x + y \\ x + 3y \end{bmatrix}$ and $S : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $S(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} 2x - 2z \\ z + 2x + y \end{bmatrix}$.

a. Find the standard matrix A of the linear transformation $T \circ S$.

b. Compute $\det(A)$.

c. Is $T \circ S$ invertible? circle your answer – no explanation is required

YES \ NO

Solution:

Solution:

Problem 3. Find all the values h for which $A = \begin{bmatrix} 2 & h & 3 \\ 1 & 0 & h \\ -1 & 2 & 1 \end{bmatrix}$ is invertible. Then for each such value h, compute A^{-1}. Your expression of A^{-1} may depend on the parameter h.

solution:

Extra Page: