Homework 9

MATH 215 (due November 14) November 7, 2022

Problem 1. 1. Prove that $\sqrt{3}$ is irrational.

2. Prove that $\sqrt{3} + 1$ is irrational.

3. Prove or disprove: the sum of any two irrational numbers is irrational.

Problem 2. Prove that for any non zero integer n, m, $gcd(n, m)$ is a linear combination of n and m. Namely, there are integers k, l such that $gcd(n, m) = kn + lm$.

[Hint: Use the Beźout Identity and another proposition we have seen in class!]

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Problem 3. Use complete induction to prove that each natural number $n > 0$ can be written as the product $n = 2^m \cdot k$, where $m \in \mathbb{N}$ and $k \in \mathbb{N}_{odd}$.

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Problem 4. 1. $f_1 : \mathbb{R} \to range(f_1)$, defined by $f_1(x) = 5x - x^2$.

Compute $f_1(1)$.

2. $f_2 : \mathbb{R} \to range(f_2)$, defined by $f_2(x) = \{x^2\}$.

   Compute $f_2(5)$.

3. $f_3 : P(\mathbb{R}) \longrightarrow range(f_3)$, defined by $f_3(x) = x \cap \mathbb{N}$.

   Compute $f_3(\{1, \pi, -1\})$ and $f_3((-\infty, 5))$.

4. $f_4 : P(\mathbb{N}) \longrightarrow range(f_4)$, defined by $f_4(x) = \begin{cases} \min(x) & 4 \in x \\ x & else \end{cases}$.

   Compute $f_4(\mathbb{N}_{even})$ and $f_4(\{n \in \mathbb{N} \mid n^2 - 2n + 1 \leq 9\})$.

5. $f_5 : P(\mathbb{R}) \longrightarrow range(f_5)$, defined by $f_5(X) = \langle X \cap \mathbb{N}, X \cap \mathbb{Z}, X \cap \mathbb{Q} \rangle$.

   Compute $f_5(\mathbb{Z})$and $f_5([-1, 1])$.

6. $f_6 : \mathbb{N} \times \mathbb{Z} \longrightarrow range(f_6)$, defined by $f_6(\langle n, m \rangle) = \{x \in \mathbb{N} \mid n < x < m\}$.

   Compute $f_6(\langle 1, 5 \rangle)$ and $f_6(\langle 1, -1 \rangle)$.

Problem 5. For each of the functions from the previous exercise, find their domain and range.