Homework 8

MATH 300 | (due April 18) | April 11, 2025

Problem 1. Prove that if $f : A \to B$, $g : B \to C$ are surjections then $g \circ f$ is a surjection.

Problem 2. Prove or disprove the following items:

1. If $f : A \to B$ is injective, then for every $X \subseteq A$, $f \upharpoonright X$ is injective.

2. If $f : A \to B$ is surjective, then for every $X \subseteq A$, $f \upharpoonright X$ is surjective.

Problem 3. Prove that if $f : A \longrightarrow B$ is a function such that for some $X \subsetneq A$, $f \restriction X : X \longrightarrow B$ is onto B, then f is not injective.

Problem 4. For each of the following functions, determine if it is injective/ surjective and prove your answer.

1. $f_1 : \mathbb{R} \to \mathbb{R}$, defined by $f_1(x) = 5x - x^2$.
2. $f_2 : \mathbb{R} \to P(\mathbb{R})$, defined by $f_2(x) = \{x^2\}$.
3. $f_3 : P(\mathbb{R}) \to P(\mathbb{N})$, defined by $f_3(x) = x \cap \mathbb{N}$.
4. $f_4 : P(\mathbb{N}) \to \mathbb{N}$, defined by $f_4(x) = \begin{cases} \min(x) & 4 \in x \\ 0 & else \end{cases}$.
5. $f_5 : P(\mathbb{R}) \to P(\mathbb{N}) \times P(\mathbb{Z}) \times P(\mathbb{Q})$, defined by
$$f_5(X) = \langle X \cap \mathbb{N}, X \cap \mathbb{Z}, X \cap \mathbb{Q} \rangle$$
6. $f_6 : \mathbb{N} \times \mathbb{Z} \to P(\mathbb{N})$, defined by $f_6(\langle n, m \rangle) = \{x \in \mathbb{N} \mid n < x < m\}$.

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Problem 5. In the following items, no proof required (just a formal definition of the functions):

1. Find an injective function $f : \mathbb{N} \to P(\mathbb{N})$.
2. Find a surjective function $f : \mathbb{Z}^2 \to \mathbb{Q}$.
3. (*Optional) Find an injective function $f : \mathbb{R} \to P(\mathbb{Q})$ [Hint: Use the density of the rationals inside the reals].
4. Find a surjective function $f : \mathbb{N} \to \mathbb{Z}$.