Homework 7

MATH 300 | (due April 11) | April 4, 2025

Problem 1.

1. $f_1 : \mathbb{R} \to codom(f_1)$, defined by $f_1(x) = \{x^2\}$.

   Compute $f_1(5)$.

2. $f_2 : P(\mathbb{N}) \to codom(f_2)$, defined by $f_2(x) = \begin{cases} \min(x) & 4 \in x \\ x & else \end{cases}$.

   Compute $f_2(\mathbb{N}_{even})$ and $f_2(\{n \in \mathbb{N} \mid n^2 - 2n + 1 \leq 9\})$.

3. $f_3 : \mathbb{N} \times \mathbb{Z} \to codom(f_3)$, defined by $f_3(\langle n, m \rangle) = \{x \in \mathbb{N} \mid n < x < m\}$.

   Compute $f_3(\langle 1, 5 \rangle)$ and $f_3(\langle 1, -1 \rangle)$.

Problem 2. For each of the functions from the previous exercise, find their domain and codomain.

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Problem 3. Define

$$f_1 : \mathbb{N} \to \mathbb{N} \times \mathbb{N}, \ \ f_1(n) = \langle n+1, n+2 \rangle$$

$$f_2 : \mathbb{N} \to \mathbb{N}, \ \ f_2(n) = n^2$$

$$f_3 : \mathbb{N} \times \mathbb{N} \to \mathbb{Z}, \ \ f_3(\langle n, m \rangle) = n - m$$

$$f_4 : \mathbb{N} \to \mathbb{N}, \ \ f_4(n) = n + 1$$

Determine if the following compositions are defined and compute them:

1. $f_1 \circ f_2$ and $f_2 \circ f_1$.
2. $f_2 \circ f_2$. and $f_3 \circ f_3$
3. $f_4 \circ f_2$ and $f_2 \circ f_4$.
4. $f_3 \circ f_1 \circ f_2$ and $f_4 \circ f_3 \circ f_2$.

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Problem 4. For a function $f : A \to B$ and $C \subseteq A$ define the *pointwise image of C by f* as

$$f''C = \{f(c) \mid c \in C\}$$

(a) Prove that if $f : A \to B$ is a function and $C \subseteq A$, then

$$(f''A) \setminus (f''C) \subseteq f''[A \setminus C].$$

(b) Give an example of sets A, B and a function $f : A \to B$ and a subset $C \subseteq A$ such that

$$(f''A) \setminus (f''C) \neq f''[A \setminus C].$$

(c) Prove that if $f : A \to B$ is an injection and $C \subseteq A$, then

$$(f''A) \setminus (f''C) = f''[A \setminus C].$$