

Homework 6

MATH 300

(due March 24)

March 14, 2025

Problem 1. Describe the set $P(\{\emptyset, \{\emptyset\}\})$ using the list principle. No proof required.

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Problem 2. Prove that for every two sets A, B the following are equivalent:

- $A \subseteq B$.
- $P(A \cup B) = P(B)$.
- $P(A) \subseteq P(B)$.

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Problem 3. Prove that for every natural number n , the number $4^n - 1$ is multiple of 3.

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Problem 4. Prove that for every natural number n , the number $17n^3 + 103n$ is multiple of 6.

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Problem 5. Prove that for every natural number n ,

$$1 + 4 + 4^2 + \cdots + 4^n = \frac{4^{n+1} - 1}{3}.$$

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Problem 6. Prove that for all odd natural numbers n , the number $2^n + 1$ is multiple of 3.

[Hint: First find an equivalent statement about all positive integers. Then use induction.]

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Next problem is optional and will not be graded for points.

Problem 7. Criticize the following obviously wrong argument:

«All horses are the same color. Specifically, every finite set of horses is monochromatic.»

Proof. We argue by induction. The statement is clearly true for sets of size 1. Assume by induction that all sets of n horses are monochromatic, and consider a set of size $n + 1$. The first n horses are all the same color. The last n horses are all the same color. Because of the overlap, this means that all $n + 1$ horses are the same color. So by induction, all finite sets of horses are all the same color, and so all horses are the same color. \square

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Next problem is optional.

Problem 8. Suppose $x \geq -1$. Prove that for all natural number n ,

$$(1 + x)^n \geq 1 + nx.$$

Next problem is optional.

Problem 9. Suppose $x \geq -1$. Prove that for all natural number n ,

$$(1 + x)^n \geq 1 + nx.$$