Homework 5

MATH 300 (due March 15) March 7, 2025

Problem 1. For any sets X, Y define $X \setminus Y = \{x \in X \mid X \setminus Y\}$.

(a) Compute $\{1,2,3\}\setminus\{1,4,7\}$, $\mathbb{N}_+\setminus\mathbb{N}_{even}$, $(1,3)\setminus[1,2)$. No proof required.

[Recall: (α, β) denote the interval of real numbers $x \in \mathbb{R}$ such that $\alpha < x < \beta$. $[\alpha, \beta)$ denote the interval of real numbers $x \in \mathbb{R}$ such that $\alpha \leq x < \beta$.]

(b) Prove the De-Morgan laws: For any sets A, B, C

(i) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

(ii) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

You can choose whether to prove by a chain of equivalences or by double inclusion.

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Problem 2. Compute the following sets. Prove two of the equalities by double inclusion.

1. $\Big\{a + b : a \in \{0, 5\}, b \in \{2, 4\}\Big\} \setminus \{7, 10\}$.
2. $(1, 3) \cup [2, 4)$
3. $\mathbb{Z} \cap [0, \infty)$
4. $\mathbb{N}_{even} \Delta \mathbb{N}_+$

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Problem 3. Prove that if $A \cap B \subseteq C$ and $x \in A \setminus C$, then $x \notin B$.

[Hint: Proof by contradiction.]