Homework 5

MATH 300 (due March 15) March 7, 2025

Problem 1. For any sets X, Y define $X \setminus Y = \{x \in X \mid X \setminus Y\}$.

(a) Compute $\{1,2,3\} \setminus \{1,4,7\}$, $\mathbb{N}_+ \setminus \mathbb{N}_{even}$, $(1,3) \setminus [1,2)$. No proof required.

[Recall: (α, β) denote the interval of real numbers $x \in \mathbb{R}$ such that $\alpha < x < \beta$. $[\alpha, \beta)$ denote the interval of real numbers $x \in \mathbb{R}$ such that $\alpha \leq x < \beta$.]

(b) Prove the De-Morgan laws: For any sets A, B, C

(i) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

(ii) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

You can choose whether to prove by a chain of equivalences or by double inclusion.

Solution (a) $\{1,2,3\} \setminus \{1,4,7\} = \{2,3\}$,
$\mathbb{N}_+ \setminus \mathbb{N}_{even} = \mathbb{N}_{odd}$,
$(1,3) \setminus [1,2) = [2,3)$
(b) (i) Let us prove this by a double inclusion:

$\subseteq$ Let $x \in Y \setminus (Y \setminus X)$. WTP $x \in X \cap Y$. By assumption, $x \in Y$ and $x \notin Y \setminus X$a and since $x \in Y$, it must follow that $x \in X$ and therefore $x \in X \cap Y$.

$\supseteq$ Let $x \in X \cap Y$. WTP $x \in Y \setminus (Y \setminus X)$. Indeed, $x \in X$ and $x \in Y$ and therefore $x \notin Y \setminus X$. By definition of difference $x \in Y \setminus (Y \setminus X)$.

Since we proved a double inclusion we conclude that $Y \setminus (Y \setminus X) = X \cap Y$.

(ii) Let us prove a double implication.

$\Rightarrow$ Suppose that $X \subseteq Y$. WTP $X \cup Y = Y$, We will prove this set equality by a double inclusion. Inclusion from right to left is clear. For the other direction, let $x \in X \cup Y$, if $x \in Y$, then we are done. If $x \in X$, then since $X \subseteq Y$ then $x \in Y$ and again we are done.

$\Leftarrow$ suppose that $X \cup Y = Y$ and let us prove that $X \subseteq Y$. Let $x \in X$, WTP $x \in Y$. It follows that $x \in X \cup Y$, and since $X \cup Y = Y$, then $x \in Y$

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Problem 2. Compute the following sets. No proof required.

1. $\Big\{a + b : a \in \{0, 5\}, b \in \{2, 4\}\Big\} \setminus \{7, 10\} = \{2, 4, 9\}$.

2. $(1, 3) \cup [2, 4) = (1, 4)$

3. $\mathbb{Z} \cap [0, \infty) = \mathbb{N}$

4. $\mathbb{N}_{even} \Delta \mathbb{N}_{+} = \{0\} \cup \mathbb{N}_{odd}$

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Problem 3. Prove that if $A \cap B \subseteq C$ and $x \in A \setminus C$, then $x \notin B$.

[Hint: Prove it by contradiction.]

Solution Suppose that $A \cap B \subseteq C$ and $x \in A \setminus C$. WTP $x \notin B$. Suppose towards a contradiction that $x \in B$. By our assumption $x \in A$ and $x \notin C$, therefore $x \in A \cap B$. Since $A \cap B \subseteq C$, then $x \in C$, this is a contradiction.