Homework 4

MATH 215 (due September 30) September 16, 2022

Problem 1. Determine which of the following statements are true. For the ones which are true, provide a proof:

1. $\{1, -1\} \in \{1, -1, \{1\}, \{-1\}\}$.
2. $7 \in \{n \in \mathbb{N} \mid |n^2 - n - 3| \leq 5\}$.
3. $1 \in \{\mathbb{N}, \mathbb{Z}, \mathbb{N}_{even}\}$.
4. $16 \in \{x \in \mathbb{N} \mid \forall y \in \mathbb{N}. y < 4 \Rightarrow y^2 + 2y < x\}$.

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Problem 2. Find a formal expression (Using the list principle or the separation principle) for the following sets:

1. The set of all integers below 100 which are are divisible by 3.

2. The set of all integers which are the successor of a power of 2.

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Problem 3. Compute the following sets using the list principle and global symbols $\mathbb{N}, \mathbb{N}_{even}, \mathbb{N}_{odd}$ and $\mathbb{Z}$. No proof in needed.

1. $\{x \in \mathbb{N} \mid \exists k \in \mathbb{N}.k + x \in \mathbb{N}_{even}\}$.
2. $\{x \in \mathbb{N} \mid x^2 + 2x - 3 = 0\}$.
3. $\{x \in \mathbb{Z} \mid \forall y \in \mathbb{N}.y < x \Rightarrow y^2 < x^2\}$

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Problem 4. Negate the following statements and proof/disprove the statement:

1. $\exists x.x < 1 \Rightarrow \exists y.y > x$.
2. $(\forall x.x > 100) \vee (\exists y.\forall x.y + x = x)$.
3. $\forall x.\forall y.x < y \Rightarrow (\exists z.x < z \wedge z < y)$.