

Homework 4

MATH 300

(due March 7)

Feb 28, 2025

Problem 1. (1) Prove that $\lim_{n \rightarrow \infty} \frac{n^2+2}{2n^2+1} = \frac{1}{2}$.

(2) Write a formula that expresses $\lim_{n \rightarrow \infty} a_n \neq L$.

(3) Prove that for every $L \in \mathbb{R}$, $\lim_{n \rightarrow \infty} (-1)^n \neq L$.

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Problem 2. (1) Prove that for every rational number $q \in \mathbb{Q}$, $\sqrt{2} \cdot q$ is irrational.

(2) Prove or disprove: the sum of irrational numbers is irrational.

(3) Prove that $\sqrt{5}$ is irrational.

(4) (optional) Formulate a conjecture for the rationality and irrationality of real numbers of the form \sqrt{n} .

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Problem 3. Determine which of the following statements are true. Prove your answer:

1. $\{1, -1\} \in \{1, -1, \{1\}, \{-1\}\}$.
2. $7 \in \{n \in \mathbb{N} \mid |n^2 - n - 3| \leq 5\}$.
3. $1 \in \{\mathbb{N}, \mathbb{Z}, \mathbb{N}_{\text{even}}\}$.
4. $16 \in \{x \in \mathbb{N} \mid \forall y \in \mathbb{N}. y < 4 \Rightarrow y^2 + 2y < x\}$.

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Problem 4. Compute the following sets using the list principle and global symbols \mathbb{N} , \mathbb{N}_{even} , \mathbb{N}_{odd} and \mathbb{Z} . No proof is needed.

1. $\{x \in \mathbb{N} \mid \exists k \in \mathbb{N}. k + x \in \mathbb{N}_{\text{even}}\}$.
2. $\{x \in \mathbb{N} \mid x^2 + 2x - 3 = 0\}$.
3. $\{x \in \mathbb{Z} \mid \forall y \in \mathbb{N}. y < x \Rightarrow y^2 < x^2\}$

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Problem 5. Find a formal expression for the following sets:

1. The set of all integers below 100 which are are divisible by 3.
2. The set of all integers which are the successor of a power of 2.
3. The set of all (exactly) two element sets of real numbers.