Homework 3

Problem 1. Prove the following equivalences (using a double implication):

An integer is divisible by 4 if and only if its last two digits form a number divisible by 4.

[Hint: Decompose $n = 100l + d$ where k, l is some integers and $0 \leq d \leq 99$. Then the number d is the last two digits.]

Solution

Problem 2. Let $x, y \in \mathbb{R}$, prove that either $x \leq y$ or there is $n \in \mathbb{N}$ such that $xn > yn + 2025$.

Solution

Problem 3. Prove that if a and b are odd integers, then $a^2 - b^2$ is a multiple of 8.

    Solution

Problem 4. Let a, b, c be integers. Prove that if $a^2 + b^2 = c^2$, then abc is even.

    Solution